

RECONCILING TONAL CONFLICTS:
MOD-7 TRANSFORMATIONS IN CHROMATIC MUSIC

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1 Mod-7 Transformations and Enharmonic Conflicts

Figure 1 gives an excerpt from César Franck’s *Symphonic Variations* for piano and orchestra. In measure 273 the music arrives on the dominant. When this chord moves into $\frac{4}{2}$ position in measure 275, it begins a dominant-seventh omnibus progression where the roots of the dominant-seventh chords creep down by minor thirds through an entire diminished-seventh arpeggio. Figure 2 shows that this example prolongs the dominant through an enharmonic progression. The common tones held between each chord on my Schenkerian sketch are not diatonically respelled, but they are respelled in the score in measure 277 for practical reasons. During the prolongation, the dominant, $C\sharp$, is theoretically respelled as $B\sharp\sharp$. While “theoretically correct”, these double sharps make the Schenkerian graph difficult to read. In order to show the prolongation as connecting two $C\sharp$ chords, as opposed to a $C\sharp$ chord and a $B\sharp\sharp$ chord, one must employ enharmonic respelling, thus sacrificing the “theoretically correct” spelling of the foreground. In this paper, I will address this type of tonal conflict between the foreground and background in nineteenth-century chromatic music. Distinguishing between $C\sharp$ and $B\sharp\sharp$ may seem only to be an intellectual exercise. Our investigation of the enharmonicism in this music, however, suggests that enharmonicism, by thwarting the diatonic background, presents fundamental structural challenges to common-practice tonality beyond ordinary chromaticism. The “correct” spelling of enharmonic progressions and the mod-7 networks that we shall use to graphically display them

will allow us to make this distinction clear.

Figure 3 displays the prolongational structure of the Franck excerpt in a transformational network, following models in Lewin's *GMIT* (1987). Some explanation of the notation used here will make the organization clear. Each chord appears in parentheses, using the note name of its root for identification. Chord quality is represented by standard notation involving capitalization of the note name and the addition of symbols for other chord features such as sevenths. For ease of reading, the chord roots are spelled as they appear in the score, even if my foreground diatonic reading would indicate a different note name for the pitch class. After the chord designation appears an Arabic numeral indicating the level of analysis. The higher the number, the closer the chords match the actual musical surface structure.

The ordered pair of numbers accompanying each arrow shows the transposition level of the harmonic progression's root motion. The first of the two numbers gives the transposition in semitones. Thus, the arrow on level two between F \sharp minor and A minor indicates that the root motion is by three semitones when moving from the F \sharp chord to the A chord. The second of the two numbers indicates how many scale steps are required to transpose the first chord to the second chord. For example, between the F \sharp and A chords, the first chord is transformed into the second by transposition up two diatonic steps (a minor third).

The ordered pair notation describing the transformations can thus be translated into standard tonal interval names. To obtain the generic interval size, add one to the second number of the pair, so that zero will be a unison, one will be a second, two will be a third, and so on. The quality of the interval—major, minor, augmented, or diminished—is encapsulated in the first number of the pair, but is dependent upon the second number. For example, an interval of a second that has one semitone is a minor second, a second that has two semitones is a major second, a second that has three semitones is an augmented second, and so forth. The first and second numbers in the ordered pair, then, come from the mod-12 and mod-7 additive groups, respectively. The mod-12 additive group, of course, is familiar

from pitch-class set theory. A great deal of literature about diatonic scale theory discusses the tonal and mathematical relationships between 12-step and 7-step scales.¹

I still represent the interval on level two between $C\sharp$ and $B\flat$ as a descending minor third (i.e. 9,5), despite its representation in the music (and in the letter names in the graph) as an augmented second. My adherence to diatonic strictness dictates that the root move by a minor third here because a progression by the chromatic interval of an augmented second makes little tonal sense. Rules for diatonic spelling in ambiguous cases are given in Table 1.

In Figure 3, the transformation between levels from the $C\sharp$ -major dominant chord at level one to the $C\sharp^7$ chord in level two is by (0,0). The ordered pair (0,0) between the levels, then, indicates that no transposition operation has taken place. The $C\sharp^7$ chord on level two descends by four minor thirds to the $C\sharp$ chord that completes the cycle. To verify that these four transformations return to a $C\sharp$ -major chord, one may add the first numbers in the ordered pairs, mod 12. Nine (from $C\sharp$ to $B\flat$) plus nine (from $B\flat$ to G) plus nine (from G to E) plus nine (from E to $C\sharp$) is thirty-six half steps. As in any twelve-tone transposition operation, the operation is addition modulo twelve. The result of the addition, thirty six, thus reduces to zero because of octave equivalence. Hence one can conclude that, at least in terms of twelve-tone equal temperament, both the $C\sharp$ chords are the same.

If there is no enharmonic shift involved, one can also expect all of the second numbers in the ordered pairs to add up to zero, when reduced modulo seven. Here, the numbers five, five, five, and five sum to twenty. Twenty mod seven is six. The music thus does not finish on the same diatonic scale step as where it began. In this mod-seven representation of scale-step intervals the transposition operation T_6 is equivalent to T_{-1} (just like in the familiar mod-twelve universe T_{11} is equivalent to T_{-1}). The $C\sharp$ is now one diatonic step lower than the $C\sharp$ chord that initiates the omnibus progression, and thus theoretically on

¹This binomial system of note or interval representation is discussed in more detail by Brinkman (1986). For more on the interaction of the diatonic and chromatic systems and complete bibliographies of the diatonic theory literature, see Agmon 1996, Clough, Engebretsen, and Kochavi 1999, Carey 1998, Santa 1999, and Jones 2002.

$B\sharp\sharp$, as was seen in Figure 2. The arrow that returns from this second “ $C\sharp$ ” back to the $C\sharp$ on level one thus cannot be (0,0). One must transpose up one diatonic step (by a diminished second) to return to the $C\sharp$ -major triad on level one. This explains the transformation by zero half steps and one scale step accompanying that arrow.

This process of reconciling surface-level transformations with the background-level chords will therefore generate all arrows in these graphs that point from a higher-numbered level back to a lower-numbered level. When chords on the two levels are diatonically different, they will be indicated by a transposition with zero as the first number and one or six as the second number. An ordered pair of (0,1) indicates that the progression has drifted down by a diatonic step, and must be shifted back up to be completely equivalent to the starting pitch level. An ordered pair of (0,6) indicates that the progression has drifted up by a diatonic step, and must be shifted back down to be completely equivalent to the starting pitch level. In the present case, the Franck variation only drifts downward diatonically, and therefore the transformation (0,6) does not appear in the mod-7 prolongational networks used here.

While the Schenkerian sketch in Figure 2 does not easily show both the change in diatonic spelling and the prolongation of the dominant, the transformational network in Figure 3 helps to clarify the relationship between the spelling and the prolongational view of the piece. Specifically, the arrow pointing from the $C\sharp$ on level two at the end of the excerpt back up to the $C\sharp$ on level one involves an enharmonic shift. This does not necessarily negate the possibility of prolongation, but rather suggests that it may be a different kind of prolongation than the kind normally seen in tonal music. The Schenkerian graph, however, clearly shows that the descending minor-third cycle is generated by an omnibus progression, a feature which is difficult to capture in the transformational network without a notation for voice exchange. The transformational network format nevertheless allows for a clear perspective on enharmonic progressions from both a foreground transformational viewpoint and a tonal background viewpoint.

Figure 1: Franck, *Symphonic Variations*, mm. 249–282.

The image displays a musical score for Franck's *Symphonic Variations*, measures 249 through 282. The score is written for piano and consists of six systems, each with a treble and bass clef staff. The key signature is three sharps (F#, C#, G#) and the time signature is 3/4. The music features a complex, rhythmic texture with frequent sixteenth-note patterns in the right hand and sustained chords or moving lines in the left hand. Performance markings include *ppp* (pianissimo) at measure 249, *pp espressivo* at measure 250, *poco cresc.* (poco crescendo) at measure 255, *ppp* at measure 258, *p espressivo* at measure 261, and *molto cresc.* (molto crescendo) at measure 264. The score is annotated with measure numbers 249, 252, 255, 258, 261, and 264 in circles at the beginning of their respective systems.

267

Musical score for measures 267-270. The piece is in G major (one sharp) and 3/4 time. Measure 267 features a treble clef with a sixteenth-note arpeggiated pattern and a bass clef with a simple accompaniment. Measure 270 includes the instruction *dimin.* and *pp* (pianissimo).

270

Musical score for measures 270-273. Measure 270 continues with *dimin.* and *pp*. Measure 271 has a *pp* marking. Measure 272 features a *pp* marking. Measure 273 includes a *pp* marking.

273

Musical score for measures 273-276. Measure 273 continues with a *pp* marking. Measure 274 has a *pp* marking. Measure 275 includes a *pp* marking. Measure 276 features a *pp* marking.

276

Musical score for measures 276-279. Measure 276 includes a *pp* marking. Measure 277 has a *pp* marking. Measure 278 features a *pp* marking. Measure 279 includes a *pp* marking.

279

Musical score for measures 279-282. Measure 279 includes the instruction *smorzando*. Measure 280 has a *smorzando* marking. Measure 281 features a *smorzando* marking. Measure 282 includes a *smorzando* marking.

282

Musical score for measures 282-285. Measure 282 includes a *smorzando* marking. Measure 283 has a *smorzando* marking. Measure 284 features a *smorzando* marking. Measure 285 includes a *smorzando* marking.

Figure 2: Schenkerian Reduction of Franck, *Symphonic Variations*, mm. 249–282.

249

f#m: i

260

iii

273

V#

Figure 3: Transformational network describing Franck, mm. 249–282.

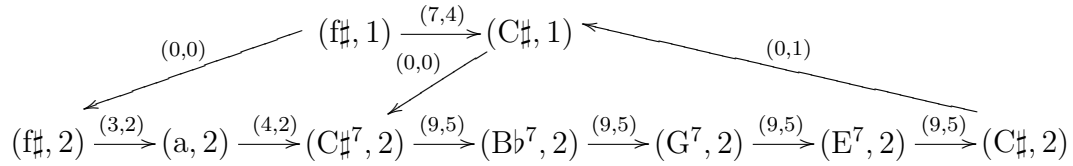


Table 1: Diatonic Spelling Rules

1. The scale degrees are to be spelled only as $\hat{1}$, $b\hat{2}$, $\natural\hat{2}$, $b\hat{3}$, $\natural\hat{3}$, $\hat{4}$, $\sharp\hat{4}$, $\hat{5}$, $b\hat{6}$, $\natural\hat{6}$, $b\hat{7}$, and $\natural\hat{7}$. If this results in a non-tertian spelling of a chord, use the procedure given in Rule 4 to decide the spelling of all members of the chord.
2. All dominant-function chords and tonicizations, including secondary dominants, secondary diminished seventh chords, altered dominants, and modulations, take the scale of the tonicized key as reference.
3. Common tones are never enharmonically respelled unless they are the result of a functional reinterpretation of a held interval between two chords (e.g. m3 becomes A2, m7 becomes A6). This may result in progressions that wander diatonically (tonic becomes \sharp VII or b bII).
4. If the chord is non-tertian and not a dominant or leading-tone chord, use the scale built on the nearest note in the triad (not the seventh) to $\hat{1}$ and $\hat{5}$ on the infinite line of fifths. In other words, find the triad member that is closest to $\hat{1}$ and $\hat{5}$ on the spiral of fifths, and spell it according to the normal diatonic scale; then spell the rest of the chord according to the intervals of the scale that is based on that note.

2 Mod-7 Transformations and Directional Tonality

With the Franck *Symphonic Variations* example, I demonstrated the use of diatonic transformations for clarifying prolongations that involve enharmonic progressions. A similar transformational approach is also useful for the analysis of other extraordinary features of nineteenth-century tonal practice. Figure 4 shows the score to Hugo Wolf's "Der Mond" from the *Italianisches Liederbuch*, Figure 5 presents a mod-7 transformation network describing the song's prolongational structure, and Figure 6 gives an English translation of the text. Figure 7 gives a middleground sketch derived from the prolongational network in Figure 5. My sketch does not account for any of the phenomenological confusion that may result from the song's ending a third below where it began. My reading nevertheless offers a strategy for listening to the piece in order to make sense of the relationship between the beginning and the end. Knowing how the song ends, one may be able to hear the opening contextually with reference to the ending key. In Figure 7, the mediant chord serves as a harmonic substitute for the opening tonic. Since there is no internal musical evidence of this substitution at the opening of the piece, it is perhaps more accurate to conceive of the piece as missing its opening. It is as though the listener has arrived late and enters the recital in the middle of the first song. This interpretation of the directional tonality seems to resonate with the effect of hearing the opening of the text. The poem's first few lines of like such a *non sequitur* in the context of the song cycle that one may wonder about the significance of this story about the moon complaining.

Figure 5 translates the prolongational structure given in Figure 7 into a series of diatonic transformations. All of the chords that are shown with open noteheads in Figure 7 appear on level one of Figure 5, though Figure 7 adds the repetition of the final V–I progression that supports the *Urblinie* descent from $\hat{3}$ to $\hat{1}$. All chords that have stems in Figure 7, except for one extra $D\flat$ chord, are chords on level two of Figure 5. (Notice the bass-clef slur from $B\flat$ in measure 6 to $G\flat$ in measure 9 that indicates that the $D\flat$ in measure 8

is a more foreground event.) In an ideal transformational graph of this type, the deepest level could contain only one chord: the tonic. In this case, however, the song's directional tonality eliminates this possibility. If one were to imply a conceptual tonic chord preceding the opening E \flat -minor chord, a closed tonal graph would obtain. Another possibility (one that I find more compelling) would be to show an arrow from the single tonic chord on level 1 to the first E \flat -minor chord on level 2 that displays a transformation other than (0,0). In this case, the ordered pair accompanying that arrow (from C \flat major to E \flat minor) would be (4,2). Whereas the Schenkerian sketch essentially normalizes the song with regard to its tonal interpretation, the transformational graph enhances one's perception of the tonal problem created by directional tonality. In this case, the analysis dramatizes one of the central issues concerning the song's tonal structure: If it is going to end in C \flat major, why does it begin in E \flat minor?

How this tonicized E \flat -minor chord functions with respect to the tonal scheme is revealed in the tonal relationships that unfold over the course of the short song. How the song clarifies the function of the first chord parallels the structure of text, which begins with a personification of the moon (in E \flat minor), then continues by blaming the narrator's lover for the moon's distress (in G \flat major), then mourns the loss of two stars from the heavens (in G \flat minor), and finally contextualizes the entire poem by revealing the metaphor of the stars as the woman's eyes (in C \flat major). The listener thus only discovers the true key of the piece when the real reason for the moon's complaint and the missing stars is disclosed.

Composers of the nineteenth century capitalize on the ambiguities of chromatic harmony in many different ways. In the present case, Wolf withholds the actual key of the piece until the end, just as the text withholds the key to the central metaphor of the poem until the end. In the Franck example, enharmonic equivalence created a confusion between different scale degrees that normally remain autonomous in tonal music. In both cases, the transformational graphs highlight the ambiguities while the Schenkerian sketches downplay

them. The strength of the Schenkerian approach is that it shows a way out of the tonal ambiguity and offers the cognitive tools for attaining a tonal hearing of the piece. When the prolongational structure is reconceived from a transformational viewpoint, the ambiguities of chromatic harmony become apparent because they create inconsistencies in the graph structure. While the diatonic transformational viewpoint is also tonally normative, it carries less theoretical baggage: No chord has any expectation of moving to any other chord in particular; there is no requirement for a monotonal analysis; there is no need to reconcile the music to an *Ursatz* structure; and post-tonal works can receive similar types of analysis. I would not wish to abandon Schenkerian analysis, even in the face of highly chromatic music. It remains the more useful interpretive tool for tonal music. Nevertheless, the mod-7 transformational viewpoint can serve to elucidate the ambiguities of chromatic harmony.

Figure 4: Wolf, “Der Mond hat eine schwere Klag’ erhoben” (1890), score.

Sehr Langsam. ♩=44.

Der Mond hat ei - ne schwe - re Klag' er - ho - ben und vor dem Herrn die Sa - che kund ge - macht;

Er wol - le nicht mehr stehn am Him - mel dro - ben, Du ha - best ihn um sei - nen Glanz ge - bracht.

Als er zu - letzt das Ster - nen - heer ge - zählt, da hab es an der vol - len Zahl ge - fehlt;

zwei von den schönsten ha - best du ent - wen - det: die bei - den Au - gen dort, die mich ver - blen - det.

p *pp* *f* *cresc.* *ppp* *pp* *sehr weich*

Figure 5: Transformational graph of Wolf, “Der Mond hat eine schwere Klag’ erhoben”

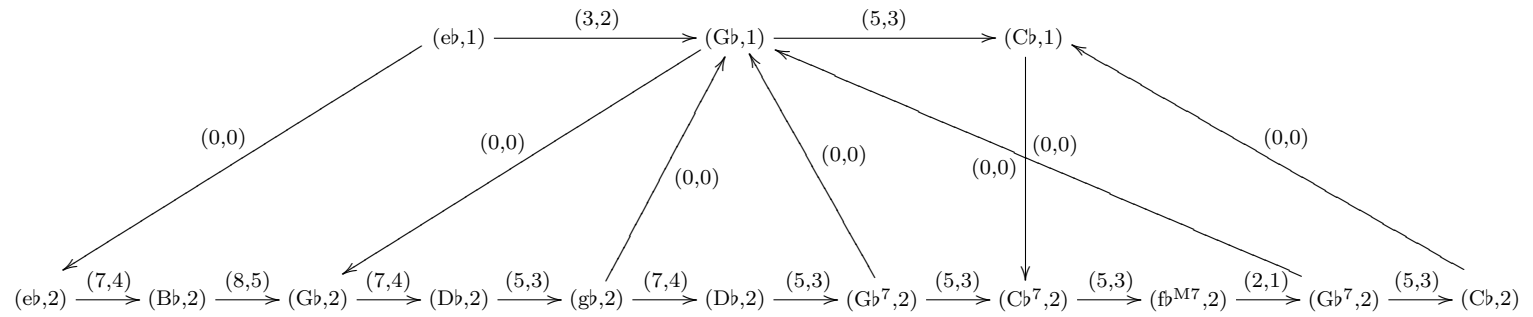


Figure 6: English Translation of Heyse, “Der Mond hat eine schwere Klag’ erhoben”

Der Mond hat eine schwere Klag’ erhoben	The moon has raised a grave complaint
Und vor dem Herrn die Sache kund gemacht;	And made the matter known unto the Lord:
Er wolle nicht mehr stehn am Himmel droben,	He no longer wants to stay in the heavens,
Du habest ihn um seinen Glanz gebracht.	For you have robbed him of his radiance.
Als er zuletzt das Sternenheer gezählt,	When he last counted the multitude of stars,
Da hab es an der vollen Zahl gefehlt;	Their full number was not complete;
Zwei von den schönsten habest du entwendet:	Two of the fairest you have stolen:
Die beiden Augen dort, die mich verblendet.	Those two eyes that have dazzled me.

Figure 7: Prolongational Sketch of Wolf, “Der Mond” based on Figure 5

The musical score for Figure 7 consists of two staves, Treble and Bass clef, with a common time signature. The melody is written in the treble clef and the bass line in the bass clef. Circled numbers 1, 5, 7, 9, 13, 15, 3, 2, and 1 are placed above the notes, indicating a sequence of pitches. Below the staves, figured bass notation and Roman numeral analysis are provided for each measure.

Figured Bass and Roman Numeral Analysis:

- Measure 1: Cb: iii
- Measure 2: V^b/iii V⁶
- Measure 3: V⁷/IV v^b
- Measure 4: V/V V⁸
- Measure 5: V/V 7
- Measure 6: I iv⁷_b
- Measure 7: V⁷
- Measure 8: I

References

- Agmon, Eytan. 1989. "A Mathematical Model of the Diatonic System." *Journal of Music Theory* 33/1: 1–25.
- . 1991. "Linear Transformations Between Cyclically Generated Chords." *Musikometrika* 3: 15–40.
- . 1996. "Coherent Tone-Systems: A Study in the Theory of Diatonicism." *Journal of Music Theory* 40/1: 39–59.
- Brinkman, Alexander. 1986. "A Binomial Representation of Pitch for Computer Processing of Musical Data." *Music Theory Spectrum* 8: 44–57.
- Carey, Norman. 1998. "Distribution Modulo 1 and Musical Scales." Ph.D. diss., University of Rochester.
- Clough, John, Nora Engebretsen, and Jonathan Kochavi. 1999. "Scales, Sets, and Interval Cycles: A Taxonomy." *Music Theory Spectrum* 21/1: 74–104.
- Jones, Evan. 2002. "Pervasive Fluency: A Contrapuntal Definition of Stability and Transience in Tonal Music." Ph.D. diss., University of Rochester.
- Karp, Cary. 1984. "A Matrix Technique for Analysing Musical Tuning." *Acustica* 54/4: 209–216.
- Lewin, David. 1987. *Generalized Musical Intervals and Transformations*. New Haven: Yale University Press.
- Samarotto, Frank. 2003. "Treading the Limits of Tonal Coherence: Transformation vs. Prolongation in Selected Works by Brahms." Paper delivered at the Twenty-Sixth Annual Meeting of the Society for Music Theory in Madison Wisconsin, November 2003.
- Santa, Matthew. 1999. "Studies in Post-Tonal Diatonicism: A Mod7 Perspective." Ph.D. diss., City University of New York.
- Stein, Deborah J. 1985. *Hugo Wolf's Lieder and Extensions of Tonality*. Ann Arbor, Michigan: UMI Research Press.