Part Writing Explained by Harmonic Function

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If you want to gain confidence in your understanding of tonal music, here is a way to look at tonal harmony that is simpler and more powerful than the traditional Roman-numeral approach. Identifying the root and quality of a chord is still an important part of analysis. In this introduction, however, we shall discover the power of thinking of chords as collections of scale degree numbers or do-re-mi solfege syllables related to the tonic. Each scale degree tends to behave in different ways depending on the type of chord in which it appears. We shall identify these behaviors using classifications called scale-degree functions.

Harmonic Functions

A scale-degree's behavior changes based on the chord's harmonic function. A harmonic function tells us how a chord behaves as part of the normal syntax of a phrase. All chords may be classified as tonic (T), subdominant (S), or dominant (D). Tonic chords are relatively stable, contain at least scale degree 1 or 3, and tend to be used at the beginning and end of a piece of music. Subdominant chords tend to push forward toward cadence points and frequently contain scale degree 4 or 6. Dominant chords are relatively unstable and contain at least scale degree 5 or 7. Dominants frequently demand resolution to a tonic-function chord, but they can also be used as a tentative point of repose.

Note that these chord designations are not chord symbols identifying the scale degree of the chord's root, but rather descriptions of how the chord is acting within a phrase. While a IV chord is a literal subdominant chord, often a composer will instead use a ii^6 chord to serve the same purpose, and we can thus label it also as functioning as a Subdominant (S). Since a chord's roman numeral will not necessarily indicate its harmonic function, a method for finding a chord's harmonic function will follow after definitions of the scale-degree functions.

Harmonic Syntax: How Phrases Are Built

Most musical phrases begin with tonic function, move through one or more subdominant chords, and then feature a cadence, either on the dominant, or using a dominant to resolve back to tonic. A typical phrase will therefore look like this: $T \rightarrow S \rightarrow D \ (\rightarrow T)$. Any chord change moving in the direction of the arrows is called an authentic progression (AP). Table 1 gives a list of the circumstances under which music may move against the authentic flow of tonality back to the previous function. This is called plagal progression (PP), and whenever it occurs, the analyst should provide the reason using one of the abbreviations in Table 1.

Table 1: Cases when plagal progression (PP) is allowed

1. Embellishing chords (passing (p) and neighboring (n) chords and pedal point (ped))
2. Plagal (PC), deceptive (DC), and half (HC) cadences

Scale-Degree Functions

Every diatonic scale degree can be used within a chord of any function. The use of some of these scale degrees, however, is restricted. Tables 2 and 3 and Figure 1 all show, in various formats, the function that each scale degree plays within each type of harmony. Students should study these charts and memorize the function of each scale degree in each harmonic context, since quickly identifying scale-degree functions will be useful for fluency in both part writing and analysis.

Table 2: Scale-Degree Functions Listed by Scale-Degree Number and Harmonic Function

Scale Degree	Tonic	Subdominant	Dominant
$\hat{1}$ *Do*	Foundation	Primary Stabilizer	Non-Chord Tone
$\hat{2}$ *Re*	Non-Chord Tone	Secondary Stabilizer	Primary Stabilizer
$\hat{3}$ *Mi/Me*	Active Tone	Chordal Dissonance	Sec. Stab./Non-Chord Tone
$\hat{4}$ *Fa*	Non-Chord Tone	Foundation	Chordal Dissonance
$\hat{5}$ *Sol*	Primary Stabilizer	Non-Chord Tone	Foundation
$\hat{6}$ *La/Le*	Secondary Stabilizer	Active Tone	Chordal Dissonance
$\hat{7}$ *Ti/Te*	Chordal Dissonance	Non-Chord Tone	Active Tone

Table 3: Membership of Each Scale-Degree Function Arranged by Harmonic Function

New Function Name	Tonic Member(s)	Subdominant	Dominant	Other Names
Foundation (F)	$\hat{1}$	$\hat{4}$	$\hat{5}$	Base (Harrison), Trigger (Quinn)
Active Tone (AT)	$\hat{3}$	$\hat{6}$	$\hat{7}$	Agent (Harrison), Trigger (Quinn)
Primary Stabilizer (PS)	$\hat{5}$	$\hat{1}$	$\hat{2}$	Associate (Harrison and Quinn)
Secondary Stabilizer (SS)	$\hat{6}$	$\hat{2}$	$\hat{3}$	Associate (Quinn)
Chordal Dissonance (CD)	$\hat{7}$	$\hat{3}$	$\hat{4}$, $\hat{6}$	Functional Dissonance (Quinn)
Non-Chord Tone (NCT)	$\hat{2}$, $\hat{4}$	$\hat{5}$, $\hat{7}$	$\hat{1}$, ($\hat{3}$)	Non-Functional Diss. (Quinn)

Foundation notes (F) are the central scale degrees of the function. Strong harmonic progressions use these notes frequently in the bass line. When writing triads in a four-voice texture, the foundation note of the current function should be doubled under ideal circumstances.

Active tones (AT) are the "color notes" that determine whether the music is major or minor. They are also the notes that help propel the music forward to the next function. Consequently, these notes are sensitive and have a specific way of resolving when the next functional category is achieved. Together, foundations and active tones are the core scale degrees of the harmonic function, and are called Harmonic Functional Triggers.

Stabilizers combine with the two functional triggers (foundation and active tone) to produce complete triads. The pitch a third above the two functional triggers is the primary stabilizer (PS), and the pitch a third below the triggers is the secondary stabilizer (SS). Most of the time, stabilizers are inert, but sometimes they may also act like dissonances in terms of how they need to be resolved. (See "How to Destabilize a Harmonic Function".)

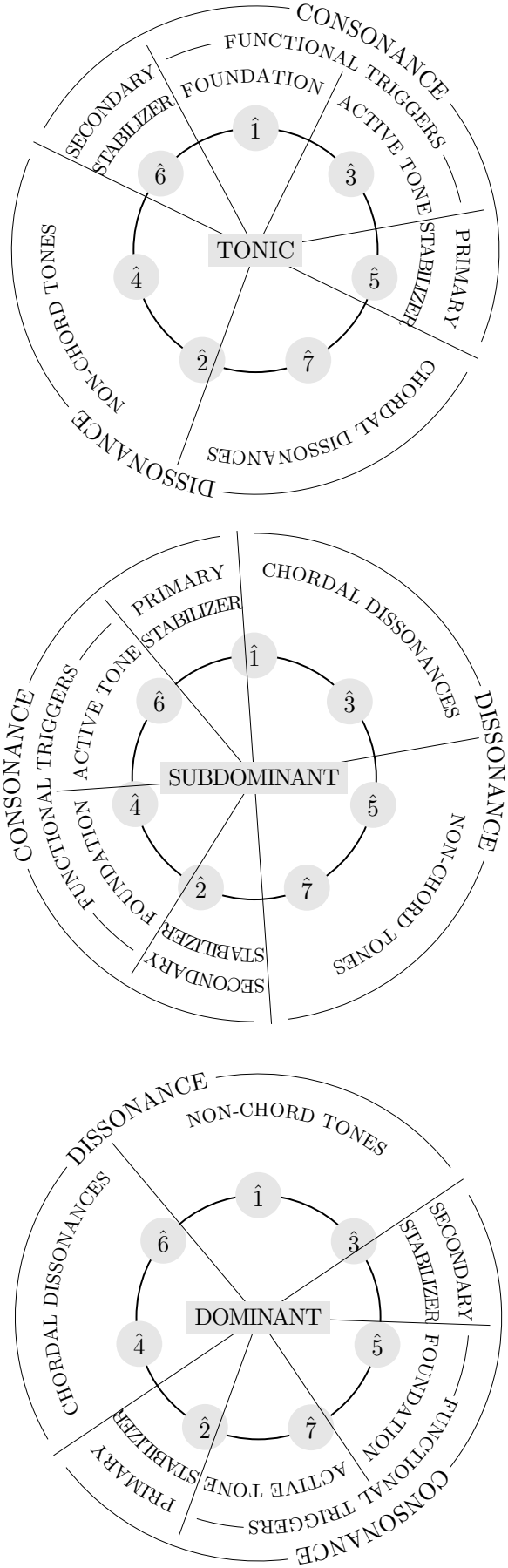

Figure 1: Chart of Scale Degree Functions, based on Quinn's 2005 Chart

Chordal dissonances (CD) are notes that clash mildly with the current harmonic function, even though they still form relatively stable chords built in thirds. All chordal dissonances resolve down by step when the music progresses authentically (AP) to the next harmonic function. If the music progresses plagally (PP), then this is not required.

Non-chord tones (NCT) cannot coexist with the stable notes in the current function. They must therefore be used only as embellishing tones. Non-chord tones resolve using traditional embellishment patterns either before or sometimes right when the next function is achieved. Embellishing melodic patterns include passing tones (PT), neighboring tones (NT), double neighbor figures (DN), appoggiaturas (App), escape tones (ET), anticipations (Ant), suspensions (Sus), and retardations (Ret).

How to Determine Harmonic Function

Before identifying a pitch's scale-degree function, you must know the harmonic function of the chord to which it belongs. To determine a chord's harmonic function, look at the scale degrees that make up the chord. All of the chord's notes should be foundations (F), active tones (AT), stabilizers (S), or chordal dissonances (CD) of its function. If there is more than one possible harmonic function for which this is true, or if there are non-chord tones in all three possible interpretations of the chord, choose the harmonic function that makes the chord contain its harmonic function's foundation and, if possible, its active tone. For the most commonly seen chords in tonal music, this means that:

- I, I^6, and iii chords have Tonic (T) function,
- ii, IV, ii^6, IV^6, and all inversions of ii^7, and IV^7 have Subdominant (S) function, and
- V, V^6, $vii^{\circ 6}$, and all inversions of V^7, and $vii^{\circ 7}$ have Dominant (D) function.

How to Destabilize a Harmonic Function

Stabilizers are pitches that combine with foundations (F) and active tones (AT) to form a triad. Adding the pitch a third above the foundation and active tone to complete the triad is stronger than adding the pitch a third below them to complete the triad. The stronger of these stabilizers is therefore called the primary stabilizer (PS), and the weaker is called the secondary stabilizer (SS).

Whenever a chord contains the weaker secondary stabilizer, the chord is said to be destabilized (x). This most commonly happens when a vi chord is used in place of a I chord, thus creating a destabilized Tonic (Tx). Likewise, a ii chord can be used as a destabilized Subdominant (Sx), and a iii chord as a destabilized Dominant (Dx). This last usage, however, is very rare.

A second way of destabilizing a function is to place the primary stabilizer in the bass voice. This destabilization is so powerful that these chords are always used as embellishing chords, and the harmonic function of the chord may even change. The most familiar example of this is the I^6_4 chord, which, despite having the scale-degree content of a Tonic-function (T) chord, often has Dominant (D) function ($D\mathit{cad}\hat{5}$).

How Stabilizers Become Chordal Dissonances or Non-Chord Tones

Sometimes a destabilized chord will contain both the primary and secondary stabilizers at the same time. When the PS and SS both appear in the same chord, they are dissonant with each other, and either the secondary stabilizer (SS) must be treated as a non-chord tone (NCT), or the primary stabilizer (PS) must be treated as a chordal dissonance (CD). In most cases, the primary stabilizer (PS) should be relabeled as a chordal dissonance (CD).

Sometimes, however, the secondary stabilizer (SS) is treated as a non-chord tone (NCT) instead. This happens more often in Dominant-function chords, whereas the primary stabilizer (PS) becomes a chordal dissonance (CD) more often in Subdominant-function chords. Furthermore, in Dominant chords, the secondary stabilizer is often treated as a NCT even when the primary stabilizer is not present, and in Subdominant chords, the primary stabilizer tends to act like a CD, even when the secondary stabilizer is not present.

How Chordal Dissonances Become Non-Chord Tones

Any scale degree that would normally be called a chordal dissonance has the potential to be used as a non-chord tone instead. Whenever this happens, the analyst should feel free to label the note as a non-chord tone (NCT), despite its chordal dissonance (CD) designation given in Table 2 (or if it is a PS in the presence of an SS). This is especially common with $\hat{6}$ in dominant function, where its label must be determined by its actual behavior in the music. For another example, any chordal dissonance on $\hat{3}$, $\hat{6}$, or $\hat{7}$ in minor that is inflected upward loses its ability to function as a chordal dissonance, and must be used as a non-chord tone.

Voice Leading

The reason for determining the harmonic and scale-degree functions for every note in a piece of music is so that the composer or analyst can then clearly see which notes have a tendency to resolve in particular ways. Voice leading is the art of writing music so that each voice flows smoothly and naturally and fulfills all of its tendencies. Knowing the classifications of these tonal behaviors will help the student to recognize when they must be obeyed, and under what circumstances they can be subverted.

Table 4 lists all of the types of scale-degree functions, and gives a voice-leading procedure for each. If a box in Table 4 contains the word "free", then there is no voice-leading expectation in the given situation. This freedom will often allow the composer to choose the remaining missing pitch that will complete the next chord given other constraints on the voice leading. The designation "CT" indicates resolution by common tone—in other words, resolution by holding or repeating the pitch into the next chord. "Step" means that the given voice has a tendency to move by major or minor second to a pitch in the next chord. Direction of resolution is free unless designated in the chart. If a voice must resolve in the "direction of inflection", this is because the note has a chromatic alteration. If the accidental makes the note a half-step higher than the diatonic pitch with the same

letter name, then it should resolve up. If the accidental has the effect of lowering the pitch a half step from the normal diatonic pitch, then it should resolve down.

Figure 2 shows all of the correct resolutions of active tones according to whether the harmonic progression is authentic (AP) or plagal (PP). This chart is encapsulated in the easy-to-remember melody *Mi Fa*, *Mi Re*, *La Sol*, *Ti Do*, which is given in Table 4. The Tonic active tone (T AT) should thus resolve *Mi* to *Fa* or *Mi* to *Re*; the Subdominant active tone (S AT) should resolve *La* to *Sol*; and the Dominant active tone (D AT) must resolve *Ti* to *Do*. This is true regardless of mode, so Tonic active tone *Me* also should resolve to *Fa* or *Re*, Subdominant active tone *Le* to *Sol*, and Dominant active tone *Te* to *Do*.

The most important rule to remember in voice leading is that dissonances (NCT, CD), active tones (AT), and chromatically altered foundations and stabilizers (Infl F, Infl S) require special resolution, but normal foundations and stabilizers do not. All scale-degree functions other than normal foundations and stabilizers are therefore called tendency tones.

Table 4: Voice-Leading Procedures

	No Function Change $T \to T, S \to S, D \to D$	Authentic Progression (AP) $T \to S, S \to D, D \to T$	Plagal Progression (PP) $T \to D, D \to S, S \to T$
NCT	DN=3rd, Others=Step	Ant.=CT, ET=Down by 3rd, Others=Step	
CD	Free	Down by Step	CT
AT	Free	Step: $M \to F$, $M \to R$, $L \to S$, $T \to D$	
Infl. F or S	CT	Step or Dim. 3rd in Direction of Inflection	
PS	Free	Step	CT
SS	Free	CT or Down by 3rd or 5th	Step
F	Free	Free	Free

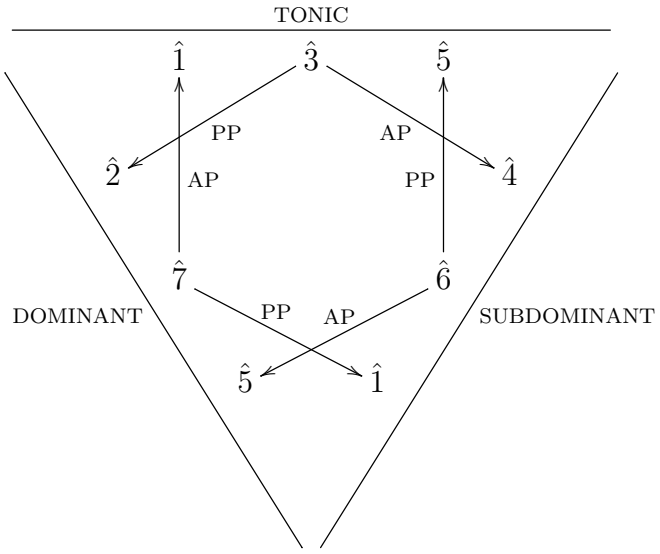

Figure 2: Chart of Active Tone Resolutions

The SATB Choral Texture

When completing part-writing exercises, the student will work under the assumption that their music should be easily performable by a four-voice mixed choir (SATB). When completing more practical projects, such as composing or arranging for a jazz ensemble, symphonic band, piano, or string quartet, some of the constraints of SATB choral writing can be relaxed. The principles of singable lines, proper voice leading, and restricting the range of each voice, however, will still assure that music for any type of ensemble will be easily performable and will sound natural to the listener. Table 5 provides the constraints to which the students should adhere in their choral writing.

Table 5: SATB Choral Writing Procedures

Vocal Ranges — *Stay between the lower and upper bounds for each part:*

S	=	C4	–	G5
A	=	F3	–	C5
T	=	C3	–	G4
B	=	F2	–	C4

Spacing — *Keep the parts close enough together with the following rules:*

Maximum allowed distance between S and A	=	1 octave
Maximum allowed distance between A and T	=	1 octave
Maximum allowed distance between T and B	=	unrestricted

Voice Crossing and Overlap — *Keep the parts from encroaching on each other's ranges by avoiding the following situations:*

- **Voice Crossing**
  - A voice is lower in pitch than the next lower voice part
  - A voice is higher in pitch than the next higher voice part
- **Voice Overlap**
  - A voice is lower in pitch than the previous note in the next lower voice part
  - A voice is higher in pitch than the previous note in the next higher voice part

Voice Independence — *The texture should always sound like it has four distinct voices moving independently:*

- **Illegal Consecutive Intervals** *occur when:*
  - Two voices a perfect unison, fifth, or octave apart (or their compound)
  - Step or skip to the same interval (or octave equivalent) in the same voices

Chord Completion — *Most chords should contain all three notes of the triad, plus a seventh if called for, with the following exception:*

- Voice leading sometimes requires the fifth to be left out of one or more chords.
  - When a root position V7 resolves to root position I, one of the two chords must leave out the fifth.

Law of the Shortest Way — *Melodies that are more conjunct are easier to sing:*

- Avoid leaps by sixths, sevenths, and chromatic intervals.
- Always prefer common tones and stepwise motion over skips, especially skips larger than a third.

Chord-Tone Doubling

Ordinarily, only triads will require doubling of one of the chord members. When the fifth must be left out of a seventh chord for voice-leading reasons, however, a chord member must then be doubled in that seventh chord. If the fifth must be left out of a triad, then more than one chord member may be doubled or a single chord member tripled. Table 6 gives a procedure for determining which note to double in a chord, and which notes never to double.

Table 6: Doubling Procedures

1. If a primary or secondary stabilizer is in the bass voice, double it.
2. If not, then if the foundation note is in the chord, double it.
3. If not, then double the bass note.
4. Voice leading may overrule the doubling indicated by the first three rules.
5. Never double dominant active tones, chordal dissonances, or inflections other than modal mixture.

When Part-Writing Preferences Are in Conflict

Balancing out good choral writing practices, voice-leading tendencies, and doubling procedures is not a simple task. The most helpful aid in accomplishing this is experience with what rules can be bent and when. Until the student has enough practice with part-writing to make consistently good decisions, the student may use a priority list as a cheat sheet. Table 7 will help the student to choose the right option whenever part-writing desiderata are at odds with each other.

Table 7: Part-Writing Priorities

1. Top Priorities	2. Important Considerations	3. Do Whenever Possible
Tendency Tone Resolutions	Law of the Shortest Way	AT Resolutions in Inner Voices
Doubling Procedure 5	Avoiding Consecutive P5s	Doubling Procedures 1–3
Avoiding Consecutive P1s & P8s	Including the 5th in Triads	Including the 5th in 7th Chords
Range, Spacing, Avoiding Crossing	Avoiding Voice Overlaps	CD Resolutions in Passing Chords

Harmonic Functional Zones

The basic phrase model presented earlier ($T \rightarrow S \rightarrow D\ (\rightarrow T)$) will always be a useful guide. It is worthwhile, however, to note that a composer can create any of the great variety of possible single-phrase chord progressions simply by expanding upon each of these harmonic functions before proceeding to the next one. In other words, each of the harmonic functions in a phrase can be thought of as functional zones, containing multiple chords. With functional zones, it is possible to write phrases as short as three chords in length or as long as thirty chords in length.

The simplest way of prolonging a functional zone is to use more chords within the same function. In this way, a Tonic functional zone could contain a I chord (T$\hat{1}$) followed by a iii chord (T$\hat{3}$), followed by a vi chord (Tx$\hat{6}$). A common feature of this type of functional expansion is bass arpeggiation.

When the function stays the same but the bass moves to another chord tone in this manner, this is called a change-of-bass prolongation. A simple example of the use of chord skips in the bass to create embellishing chords would be a Tonic zone consisting of inversions of the I chord: I T$\hat{1}$ $\rightarrow$ I^6 T*arp*$\hat{3}$ $\rightarrow$ I^6_4 T*arp*$\hat{5}$ $\rightarrow$ I T$\hat{1}$.

It is uncommon to end a functional zone with a chord whose bass note is something other than the function's foundation or active tone. It is even more unusual to begin a functional zone without the foundation or active tone in the bass. This is therefore strongly discouraged.

Just like a composer may wish to fill in a melodic skip using a passing tone, a composer may wish to fill in the skips in the bass arpeggiation with harmonized passing tones. This type of embellishing chord is called a passing chord and the analyst should label these with the letter P. Some common examples of a tonic functional zone expanded using a passing chord include I T$\hat{1}$ $\rightarrow$ $vii^{\circ 6}$ Dp$\hat{2}$ $\rightarrow$ I^6 T$\hat{3}$; and I T$\hat{1}$ $\rightarrow$ V^6 Dp$\hat{7}$ $\rightarrow$ vi Tx$\hat{6}$.

If bass passing tones can become passing chords, then so can neighboring tones in the bass. Neighbor chords are analyzed with the letter N. Some examples of neighbor chords within a tonic functional zone include I T$\hat{1}$ $\rightarrow$ V^6_5 Dn$\hat{7}$ $\rightarrow$ I T$\hat{1}$; and I^6 T$\hat{3}$ $\rightarrow$ V^4_2 Dn$\hat{4}$ $\rightarrow$ I^6 T$\hat{3}$. Incomplete and double neighbor chords are possible as well.

Passing chords can also be used to smooth the transition to the next functional zone. Here are two examples: I T$\hat{1}$ $\rightarrow$ V^6 Dp$\hat{7}$ $\rightarrow$ IV^6 S$\hat{6}$; and ii S$\hat{2}$ $\rightarrow$ I Tp$\hat{1}$ $\rightarrow$ V^6_5 D$\hat{7}$. Although these are somewhat uncommon, it is important to be able to recognize them when they occur.

Bass pedal point is a common technique for expanding a single chord into a longer functional zone. These can be as simple as pedal six-four chords (I T$\hat{1}$ $\rightarrow$ IV^6_4 S*ped*$\hat{1}$ $\rightarrow$ I T$\hat{1}$) and cadential six-four chords (I^6_4 D*cad*$\hat{5}$ $\rightarrow$ V D$\hat{5}$), or as complex as complete functional progressions above a sustained bass note.

Finally, tonic functional zones can be prolonged using subsidiary harmonic progressions (also called embedded phrase models). Subsidiary harmonic progressions are complete T–S–D–T cycles that are weakened by the use of a non-cadential ending (such as D$\hat{7}$ $\rightarrow$ T$\hat{1}$; D$\hat{5}$ $\rightarrow$ T$\hat{3}$; or D$\hat{5}$ $\rightarrow$ Tx$\hat{6}$) and usually also stepwise bass motion within the harmonic functional zone.

It is worthwhile to note that frequently when functional zones are prolonged, the bass line is static or stepwise and the prolonging chords are not in root position. This makes it easier to recognize the difference between contrapuntal progressions, which tend to prolong a functional zone, and functional progressions, which lead strongly toward a cadence. Contrapuntal progressions tend to be more stepwise, especially in the bass, while functional progressions feature strong bass motion from the foundation of one harmonic function to the foundation of the next.

Because functional zones are prolonged using conjunct voice leading, the use of embellishing chords makes stepwise voice leading in all parts of paramount importance. Because of this, certain tendency tones may sometimes be ignored when resolving a passing or neighboring chord. Specifically, the chordal dissonance may sometimes resolve up by step instead of down, even when the passing or neighboring chord can accurately be said to move by authentic progression (AP) to the harmony that follows it. This is most commonly seen in the progressions I T$\hat{1}$ $\rightarrow$ $vii^{\circ 6}$ Dp$\hat{2}$ $\rightarrow$ I^6 T$\hat{3}$; and I T$\hat{1}$ $\rightarrow$ V^4_3 Dp$\hat{2}$ $\rightarrow$ I^6 T$\hat{3}$.

Part-Writing Assessment

Students who are given part-writing exercises to complete should take care to check their work for accuracy and quality in the six areas listed in Table 8. Since the teacher will use these categories to assess the student's work, the student is therefore advised to evaluate their own attainment of these goals in their own work before handing it in. Revision may be necessary if the student finds errors in their writing or is not satisfied with the quality of their music's style, melodic writing, or voice leading. Table 8 also gives a list of errors to look for, and notations that a teacher may use to mark shortcomings in the student's part writing.

Table 8: Notations for Part-Writing Errors

- Correct pitches and rhythms

  WN = Wrong note
  WC = Wrong chord
  WINV = Wrong Inversion

- Idiomatic SATB choral writing

  r = Voice range exceeded
  L = Difficult leap in voice part
  S = Spacing error
  VC = Voices Crossed
  VO = Voice Overlap
  P1, P5, P8 = Consecutive perfect unisons, fifths, or octaves
  inc = Incomplete chord written when a complete chord was possible
  INC = Crucial note missing from chord

- Harmonic progression

  WP = Weak Progression

- Doubling

  d = Weak doubling
  D = Illegal Doubling

- Voice leading

  R = Tendency tone not resolved properly

- Analysis of work

  ? = Missing analysis
  X = Incorrect analysis
  Type? = Missing justification for irregular progression or voice leading

How to Label Your Composition or Analysis

It is important to identify every element in a piece of music using a music theory designation. Each phrase of music ends in a cadence. The analyst must therefore label every cadence in the music. Cadences are analyzed according to the type of cadence. Cadence types include: perfect authentic cadence (PAC), imperfect authentic cadence (IAC), half cadence (HC), tonicized half cadence (THC), deceptive cadence (DC), or plagal cadence (PC).

The analyst must identify the root and quality of every chord, both with and without reference to the prevailing key. This requires the use of both lead-sheet chord symbols (also known as pop chord symbols) above the music, and Roman numerals with figured bass numbers below the music. Most importantly, any complete harmonic analysis must include functional-bass chord symbols (i.e. harmonic functions plus bass scale-degree numbers) below the staff. Because Roman numerals and functional-bass symbols are dependent on the music's key for interpretation, the analyst must provide the key and mode of the music to the left of the string of analysis symbols. Once the harmonic function of each chord has been determined, the student should then label each note with its scale-degree function and arrows showing all resolution tendencies.

All part-writing and analysis exercises may be labeled using the abbreviations for each function. Figure 3 gives an example of some of these labels. The functional-bass analysis of each chord is given using a single letter abbvreviation of its harmonic function followed by the bass scale-degree number. For secondary dominants and modulations, add a slash and the roman numeral of the key in which the chord is functioning. Further, any non-chord tone (NCT), chordal dissonance (CD), active tone (AT), or inflected foundation or stabilizer (Infl F, Infl S) should be labeled with an arrow that shows its direction of resolution. (Even though they are not technically considered to be tendency tones, resolution arrows can be added for stabilizers (PS and SS) as well.)

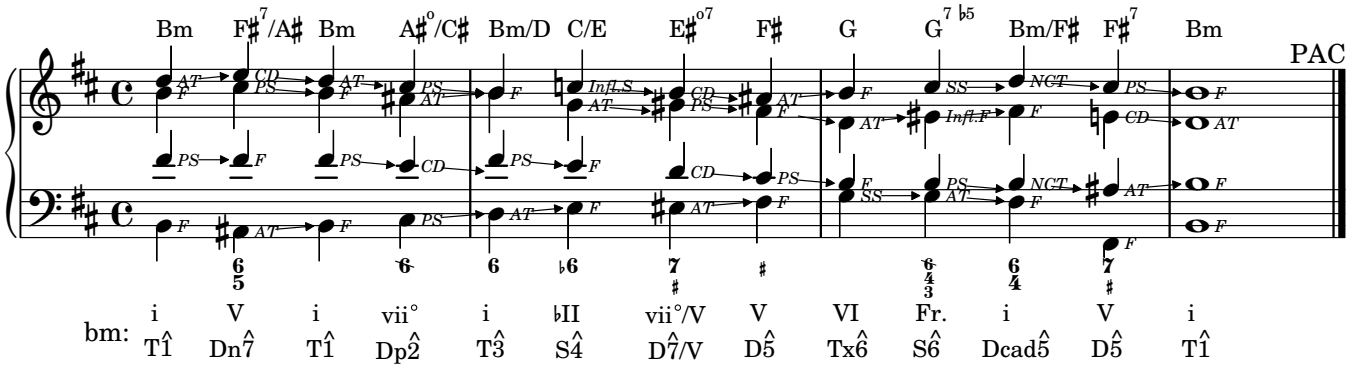

Figure 3: Example of a Complete Functional-Bass Analysis

References

Aarden, Bret, and Paul T. von Hippel. 2004. "Rules for Chord Doubling (and Spacing): Which Ones Do We Need?" *Music Theory Online*, vol. 10/2. <http://www.mtosmt.org/issues/mto.04.10.2/mto.04.10.2.aarden_hippel_frames.html>.

Aldwell, Edward, and Carl Schachter. 2010. *Harmony and Voice Leading*. 4th ed. Cengage Learning.

Burstein, L. Poundie, and Joseph N. Straus. 2016. *Concise Introduction to Tonal Harmony*. W. W. Norton.

Harrison, Daniel G. 1994. *Harmonic Function in Chromatic Music: A Renewed Dualist Theory and an Account of Its Precedents*. University of Chicago Press.

Kostka, Stefan, and Dorothy Payne. 1984. *Tonal Harmony, with an Introduction to Twentieth-Century Music*. 7th ed., 2013. New York: A. A. Knopf.

Mavromatis, Panayotis. 2009. "HMM Analysis of Musical Structure: Identification of Hidden Variables Through Topology-Sensitive Model Selection." *Communications in Computer and Information Science: Mathematics and Computation in Music* 38: 205–217.

McHose, Allen Irvine. 1951. *Basic Principles of the Technique of 18th and 19th Century Composition*. New York: Appleton-Century-Crofts, Inc.

Motte, Diether de la. 1976. *The Study of Harmony: A Historical Perspective*. Translated by J. L. Prater. Dubuque, Iowa: W. Brown, 1991.

Quinn, Ian. 2005. "Harmonic Function Without Primary Triads." Paper delivered at the Twenty-Eighth Annual Meeting of the Society for Music Theory in Boston, November 2005.

Quinn, Ian, and Panayotis Mavromatis. 2011. "Voice-Leading Prototypes and Harmonic Function in Two Chorale Corpora." Edited by Carlos Agon, Moreno Andreatta, Gérard Assayag, Emmanuel Amiot, Jean Bresson, and John Mandereau, *Mathematics and Computation in Music: Third International Conference, MCM 2011, Paris, France, June 15–17, 2011. Proceedings*. Springer Berlin Heidelberg.

Quinn, Ian, and Christopher White. 2013. "Expanding Notions of Harmonic Function Through a Corpus Analysis of the Bach Chorales." Paper delivered at the Annual Meeting of the Society for Music Theory.

———. 2015. "A Corpus-Based Model of Harmonic Function in Popular Music." Paper delivered at the Annual Meeting of the Society for Music Theory.

Shaffer, Kris, and Robin Wharton, eds. 2014. *Open Music Theory*. Hybrid Pedagogy Publishing. <http://openmusictheory.com/>.

Swinden, Kevin J. 2005. "When Functions Collide: Aspects of Plural Function in Chromatic Music." *Music Theory Spectrum* 27/2: 249–282.